1

Classical Mechanics 1

1.1 Dimensions and units

The SI units are split into two groups: the 7 base units and the 22 derived units. The base quantities are the set of units from which all other units can be derived. Table (1.1) shows the 7 base physical dimensions, each corresponding to an SI base unit.

The *dimension* of a quantity is not the same as the *units* of a quantity. The dimension of a quantity is a more fundamental than its units. A quantity may have the dimension of length, but could be given in units of m, au, cm, ...

When we combine SI base units, we form the SI derived units. There are 22 derived units which are named, but many of them are rarely used in physics. Table (1.2) lists the most common derived units in physics.

How do we determine the dimensions of a quantity? Let's take say we wanted to determine the dimension symbol for the force.

Quantity	Symbol	Base unit
time	T	s
length	L	m
mass	M	kg
electric current	I	A
thermodynamic temperature	Θ	K
amount of substance	N	mol
luminous intensity	J	cd

Table 1.1: The SI base quantites, alongside their dimension symbol and SI unit.

Determine the dimension symbols for a force.

We choose a simple equation containing a force and re-arrange if necessary. Let's choose Newton's second law

$$F = ma, \tag{1.1}$$

where F is the force, m is the mass and a is the acceleration. We denote the dimensions of a quantity with square brackets $[\,\cdot\,]$. The dimensions of the force are therefore

$$[F] = [ma] = M \cdot LT^{-2}. \tag{1.2}$$

[We can substitute in a unit system, for example the SI base units, to say that an example of the *units* of force are kg m s^{-2}].

You can make use of the fact that the transcendental functions (e.g., sin, tanh, ln, exp, ...) must have a *dimensionless* argument — the dimensions are unity.

Determine the dimension symbols for the linear attenuation coefficient in $I = I_0 e^{-\mu x}$.

Regardless of what the quantities I and I_0 represent, we can perform dimensional analysis on the argument of the exponential to determine $[\mu]$.

Given that x represents a distance,

$$[\mu x] = 1 \implies [\mu] = \frac{1}{[x]} = L^{-1}. \tag{1.3}$$

The dimensions of μ are inverse length. An example of units for μ are cm^{-1}.

When units are typeset, it is best practice to leave a space between the number and the unit, and a space between each unit. Unit prefixes (e.g., M, p, ...) do not have a space after them. Units must not be italicised.

Quantity	**Equation**	**Symbol**	**Base unit**
hertz	$f = 1/T$	T^{-1}	s^{-1}
newton	$F = ma$	$M \cdot LT^{-2}$	$\mathrm{kg\,m\,s}^{-2}$
pascal			
joule			
watt			
coulomb			
volt			
farad			
ohm			

Table 1.2: 9 of the 22 SI derived units.

1.2 Significant figures

A calculator or a computer can calculate quantities to arbitrary precision, but this almost always has no real-world meaning. We need to be careful in our interpretation of a numerical answer, and that almost always is an exercise in quoting your answers to the correct precision.

You should almost always give your answer to the same precision as the least precise value or measurement that you have. During calculations, you should retain one further significant figure to avoid rounding errors.

Leading zeros are not significant figures. Trailing zeros are significant figures. This means that `0.0000001` has only one significant figure whilst `1000000` has 6 significant figures. We must use significant figures and unit prefixes to ensure our final answer has *physical* meaning, and is not just some arbitrary number our calculator or computer returns to us.

How many centimetres does a photon travel in 0.9 ns?

a) 27 cm	b) 30 cm
c) 27×10^0 cm	d) 30×10^0 cm

Worked example

When you are presenting a result from a calculation or experiment, you must make sure you i) give your answer to the correct precision and ii) give the correct units. You will lose a mark every time you make these mistakes.

Calculate the molar gas constant, R, given experimental measurements of the variables $p = 3.0$ atm, $V = 6.2$ L, $n = 0.75$ mol and $T = 310$ K. [You may find the following conversions useful: 1.00 atm = 101 kPa, 1.00 L = 0.00100 m^3**.]**

We calculate the molar gas constant R via the ideal gas equation

$$pV = nRT, \tag{1.4}$$

where p is the absolute pressure of the gas, V is the volume of which it occupies, n is the amount of substance in the sample and T is the absolute temperature of the system.

We can compute R as

$$R = \frac{pV}{nT} = \frac{3 \times 101 \times 10^3 \times 6.2 \times 0.00100}{0.75 \times 310} = 8.08. \tag{1.5}$$

We round R to 2 significant figures — the precision of our least precise variable. Therefore, $R = 8.1$.

What are the dimensions of R?

$$[R] = \frac{[pV]}{[nT]} = \frac{ML^{-1}T^{-2} \cdot L^3}{N \cdot \Theta} = ML^2T^{-2}N^{-1}\Theta^{-1}. \tag{1.6}$$

So the units of R in our unit system is kg m^2 s^{-2} mol^{-1} K^{-1}. And so

$$R = 8.1\,\text{kg}\,\text{m}^2\,\text{s}^{-2}\,\text{mol}^{-1}\,\text{K}^{-1}, \tag{1.7}$$

or we can recognise that 1 J = 1 kg m^2 s^{-2}, to re-write the molar gas constant as

$$R = 8.1\,\text{J}\,\text{mol}^{-1}\,\text{K}^{-1}. \tag{1.8}$$

1.3 Applications of dimensional analysis

You can also use dimensional analysis to find equations of physical quantities.

A wind turbine converts kinetic energy in the wind to electric energy. The rate at which it produces energy should depend on wind speed and the size of its blades. Are any other dimensional variables it should depend on? Find an equation for its power output, P.

The power output of the turbine is a function of the speed of the wind v, the length of the blades L and the density of the air ρ flowing past the blades

$$P \equiv P(\rho, v, L). \tag{1.9}$$

Dimensional analysis can tell us that the equation for P must be of the form

$$P \propto \rho^{\alpha} v^{\beta} L^{\gamma}, \tag{1.10}$$

where $\alpha, \beta, \gamma \in \mathbb{Q}$ (some quantities may be divided or multiplied).

The dimensions must balance on either side of the proportionality statement. The dimensions of power are energy divided by time, i.e., $[P] = ML^2T^{-2}/T = ML^2T^{-3}$. The dimensions of density[a] are mass divided by volume, $[\rho] = ML^{-3}$. The dimensions of velocity are length divided by time, $[v] = LT^{-1}$. The dimensions of length is $[L] = L$. Thus,

$$ML^2T^{-3} = (ML^{-3})^{\alpha} \cdot (LT^{-1})^{\beta} \cdot L^{\gamma}, \tag{1.11}$$

and the values of α, β, γ for the equation to balance (the same number of M, L and T on either side of the equals sign) must be $\alpha = 1$, $\beta = 3$ and $\gamma = 2$ so

$$P = k\rho v^3 L^2, \tag{1.12}$$

where k is a (dimension*less*) constant of proportionality.

[a] Care must be taken if you are discussing *linear* density (e.g., in electromagnetism) which has dimensions of mass per unit length.