11

Electromagnetism 1

11.1 Linear superposition of charges

The linear superposition of charges states that every charge in space creates a local electric field at that point, independent of other charges, and the resultant electric field is the summation of the electric field due to the individual charges. Figure (11.1) shows a distribution of four charges, of the order of μC. Let us try and calculate the magnitude of the electric field at location C. By the linear superposition of electric fields,

$$\mathbf{E}_C = \mathbf{E}_{A-C} + \mathbf{E}_{B-C} + \mathbf{E}_{C-C} + \mathbf{E}_{D-C}, \tag{11.1}$$

where the notation $\mathbf{E}_{A-C}$ reads as 'the electric field at location C due to the charge at A'.

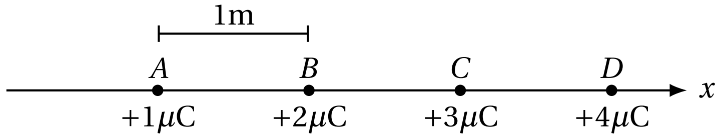

Figure 11.1: A one dimensional distribution of charges is shown. The magnitude of the charge increases from left to right. Each charge is separated by a distance of 1 m.

Study figure (11.1). What is the magnitude of the electric field at C, due to charge C, i.e., $|\mathbf{E}_{C-C}|$?

a) 0	b) kq
c) ∞	d) not enough info (yet!)

Determine the magnitude of the electric field at C.

We must remember that the electric field due to a point charge (or objects with spherical symmetry) is given by

$$\mathbf{E}(\mathbf{r}) = \frac{kQ}{r^2}\hat{\mathbf{e}}_r, \tag{11.2}$$

where $k = 1/4\pi\varepsilon_0$ is the electrostatic constant and $\hat{\mathbf{e}}_r$ is a unit vector pointing radially away from the charge.

At the point C, the charge experiences the field $\mathbf{E}_{C-C}$, but does not contribute to it. Equation (11.2) tells us that the electric field is infinity at this point (as $r \to 0$). However, since the charge does not contribute to the total electric field at this point, $\mathbf{E}_{C-C} = 0$.

The contribution of the electric fields due to the other charges are simpler to deal with. We must pay attention to the sign of the electric field due to the charge at D—its position unit vector $\hat{\mathbf{e}}_r$ acts towards C so it must be negative.

From equation (11.1), and using the data from figure (11.1),

$$\begin{aligned}
\mathbf{E}_C &= \frac{k}{Q_A} r_{A-C}^2 \hat{\mathbf{e}}_r + \frac{k}{Q_B} r_{B-C}^2 \hat{\mathbf{e}}_r + \mathbf{0} - \frac{k}{Q_D} r_{D-C}^2 \hat{\mathbf{e}}_r \\
\implies |\mathbf{E}_C| &= k \left[\frac{10^{-6}}{2^2} + \frac{2 \times 10^{-6}}{1^2} + 0 - \frac{4 \times 10^{-6}}{1^2} \right] \\
&= -\frac{7}{4} k \, \mu\mathrm{N\,C^{-1}} \\
&= -2 \times 10^4 \,\mathrm{N\,C^{-1}}.
\end{aligned} \tag{11.3}$$

This result makes sense—the dominant electric field is due to the charge at D.

What is the force on the charge C? This is simply Coulomb's law

$$|\mathbf{F}| = k \frac{|\mathbf{q_1} \cdot \mathbf{q_2}|}{r^2}, \tag{11.4}$$

or,

$$|\mathbf{F}| = q_C |\mathbf{E}_C| = -6 \times 10^4 \,\mathrm{N} \tag{11.5}$$

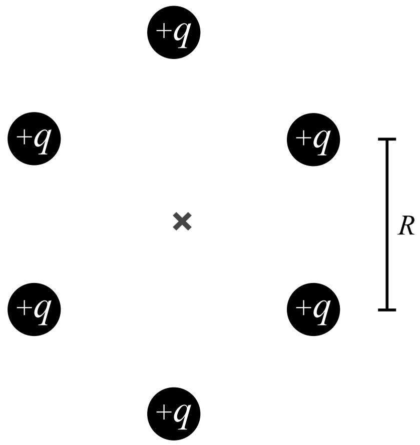

Figure 11.2: A hexagonal distribution of positive point charges of charge $+q$. The point charges lie on the vertices of the hexagon of side length R.

Let us now apply the principle of the linear superposition of electric fields to another problem. Consider the hexagonal distribution of charges in figure (11.2).

Study figure (11.2). What is the net electric field at the centre ($\times$) in this system?

a) $\mathbf{E} = kq(\hat{\mathbf{e}}_x + \hat{\mathbf{e}}_y)$	b) $\mathbf{E} = \mathbf{0}$
c) $\mathbf{E} = kq\,\hat{\mathbf{e}}_y$	d) $\mathbf{E} = 0\,\hat{\mathbf{e}}_y$

By symmetry, all charges cancel, so the net force at the centre is $\mathbf{E} = \mathbf{0}$.

Suppose the bottom charge was removed in figure (11.2).
What is the net electric field at the centre $(\times)$ in this system?

a) $\mathbf{E} = \frac{2kq}{R^2}\,\hat{\mathbf{e}}_y$	b) $\mathbf{E} = -\frac{2kq}{R^2}\,\hat{\mathbf{e}}_y$
c) $\mathbf{E} = \frac{kq}{R^2}\,\hat{\mathbf{e}}_y$	d) $\mathbf{E} = -\frac{kq}{R^2}\,\hat{\mathbf{e}}_y$

By symmetry, all charges on the left and right, leaving only the contribution from the top charge, which points towards $\times$. The net electric field is $\mathbf{E} = -\frac{kq}{R^2}\,\hat{\mathbf{e}}_y$.

11.2 Sketching field lines

Sketching field lines is a common source of mistakes in examinations. Field line sketches require:

1. correct directions (positive charge to negative charge),
2. field lines to direct radially at the charges,
3. no intersections and
4. the density of lines to be proportional to the magnitude field at that point.

Strictly speaking, the density of lines is only proportional to the magnitude of the field in 3D. This is not true in 2D, but we must still convey the fact that there is a larger density of lines where the field is stronger. This usually means drawing double the number of field lines for a charge of magnitude $2q$ compared to a charge of magnitude q.