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Classical Mechanics 2

2.1 Order of magnitude estimations

Sometimes we do not know precise values for quantities in our calculations. In these circumstances, we may only be interested in whether our answer from our lab or numerical experiments is roughly correct. We instead care more about the powers of ten involved in the calculations. Constants such as 1.23 attached to the power of ten become meaningless in such estimations.

Powers of 10 vs orders of magnitude

In $a \times 10^b$, a is a constant, 10^b is the power of ten and b is the order of magnitude. Taking the base 10 logarithm of a number and rounding it to the nearest integer will give you its order of magnitude b. For example, for the number 3120, one has $\log_{10} 3120 = 3.49...$ so it is of the order of 10^3; for the number 3163, one has $\log_{10} 3163 = 3.50...$ so it is of the order of 10^4. This is known as logarithmic rounding

Alternatively (and more quickly), you can compare the constant a to $\sqrt{10} \approx 3.1623$—if the constant is less than $\sqrt{10}$, you round down to the nearest power of ten, and if the constant is greater than $\sqrt{10}$, you round up to the nearest power of ten.

We use the phrase 'order of magnitude estimation' quite loosely, however, and usually take this to mean 'power of ten estimation'.

Order of magnitude estimation

You will develop an intuition for the order of physical quantities. This will make order of magnitude estimations very easy. The idea is to make quick calculations without the aid of a calculator, and rather by using the rules of indicies.

For example: the average human walking speed is of the order of 10^0 m s^{-1}, the average mass of a human is of the order of 10^2 kg and the mass of the Earth is of the order of 10^{25} kg. We can also use the tilde ~ in this context to mean 'of the order of', for example, the mass of the sun $m_\odot \sim 10^{30}$ kg.

A final answer obtained using orders of magnitude estimation must be quoted with the ~ sign.

2.2 Relative motion

Relative motion simply means that there are different values of position, velocity and (in non-inertial frames) acceleration in different reference frames. A reference frame is simply a set of geometric axes attached to an object which may or may not be moving. Let's look at the stationary case first.

Figure (2.1) shows two two-dimensional frames: i) a lab frame L and ii) my frame J. The 'lab' frame describes anything which we generally take to be stationary (a lab, a building, the Earth, etcetera). My frame also has a set of axes attached to it, and those axes move around with me as I move — I am always at the origin of my frame.

We can form position vectors $\mathbf{r}$ between the lab frame and Bob, the lab frame and I and Bob and I. These form a vector triangle satisfying

$$\mathbf{r}_{B-L} = \mathbf{r}_{B-J} + \mathbf{r}_{J-L}. \tag{2.1}$$

If the lab frame, Bob and I were to begin moving, then the first and second time derivatives of equation (2.1) hold:

$$\mathbf{v}_{B-L} = \mathbf{v}_{B-J} + \mathbf{v}_{J-L}, \tag{2.2}$$

and

$$\mathbf{a}_{B-L} = \mathbf{a}_{B-J} + \mathbf{a}_{J-L}. \tag{2.3}$$

It is equation (2.2) that you will use the most in relative motion problems. When the acceleration is 0, the motion is *uniform*.

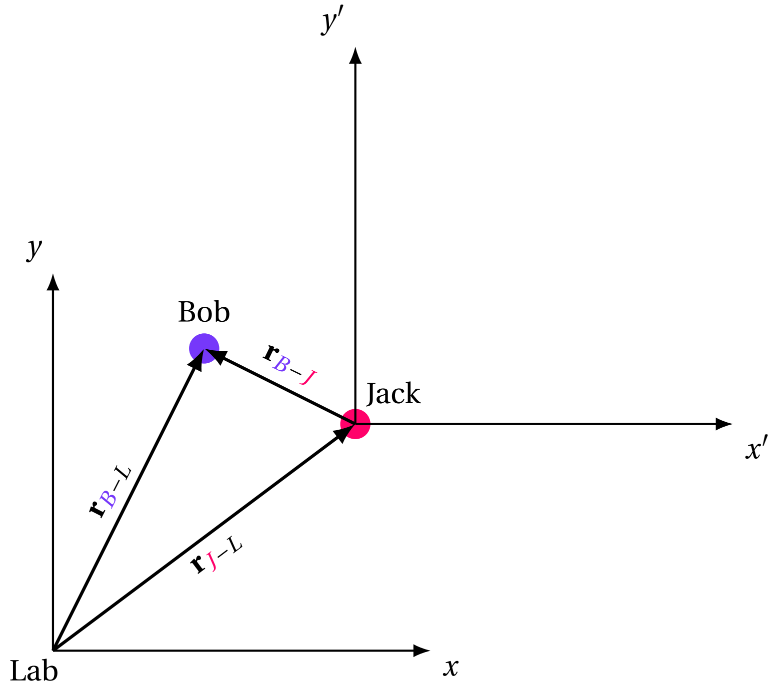

Figure 2.1: Transformation of a frame S to another frame S', moving at a velocity v with respect to the stationary frame.