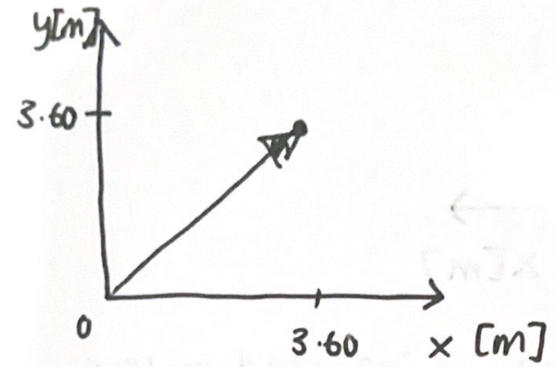
Mech 1.2

Consider the displacement of the cutting tool from $(0,0)$ to $(3.6, 3.6)$ m.



the work done on the tool by a force

$$\underline{F} = -\alpha x y^2 \hat{\underline{e}}_y$$

is the line integral

$$W = \int_{y_1}^{y_2} \underline{F} \cdot d\underline{l},$$

where the line element

$$d\underline{l} = dx\,\hat{\underline{e}}_x + dy\,\hat{\underline{e}}_y .$$

Since there's no force in the $\hat{\underline{e}}_x$ dir^n, the integrand becomes, along path $x=y$,

$$\underline{F} \cdot d\underline{l} = -\alpha y^3 dy$$

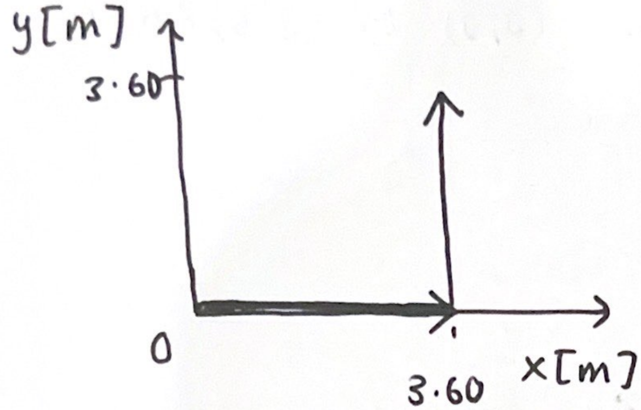
from $y_1 = 0 \rightarrow y_2 = 3.60$ m, the work done is

$$\Rightarrow W = -\int_0^{3.60} \alpha y^3 dy = \frac{\alpha}{4} (3.60^4 - 0^4)$$

$$\Rightarrow W = -\frac{1}{4}(2.40)(3.60)^4 = -101 \text{ J} .$$

The work done along the line $y=x$ is $W = -101$ J.

b) the tool is moved along $\hat{e}_x$ then along $\hat{e}_y$, as per the figure below



Along $\hat{e}_x$, the force is perpendicular along the displacement at every point, so there is no work done.

Along $\hat{e}_y$, $\underline{F} \cdot d\underline{\ell} = -\alpha x y^2 dy$, the work done is

$$\Rightarrow W = -\alpha x \int_0^{3.60} y^2 dy = -\alpha x \frac{1}{3}(3.60^3 - 0^3)$$

$$= -2.40 \cdot 3.60 \cdot \frac{1}{3} \cdot 3.60^3 = -134J$$

The work done along the x-axis is 0J.
The work done along $\hat{e}_y$ is $-134J$.

c) the work done along the two paths is different (despite starting & ending at the same point) so the force is non-conservative.