Mech 2.1

1.1 the angular displacement of the motor shaft is

$$\theta(t) = 241t - 20.2t^2 - 1.51t^3.$$

the angular velocity is

$$\omega(t) = \dot{\theta}(t) = 241 - 40.4t - 4.53t^2$$

and the angular acceleration is

$$\alpha(t) = \ddot{\theta}(t) = -40.4 - 9.06t.$$

1.1. the angular velocity is zero at

$$\omega(t) = 241 - 40.4t - 4.53t^2 = 0$$

$$\Rightarrow t = \cdots = 4.09\ s$$

1.2. the angular acceleration at the instant when $\omega = 0$ is

$$\alpha(4.09) = -40.4 - 9.06(4.09)$$

$$\Rightarrow \alpha(4.09) = -77.5\ rad\ s^{-2}.$$

1.3. the angular displacement after 4.09 s is

$$\theta(4.09) = 241(4.09) - 20.2(4.09)^2 - 151(4.09)^3$$

$$= 544.5\ rad.$$

which implies that the number of revolutions is $\frac{\theta(4.09)}{2\pi} = \frac{544.5}{2\pi} = 86.7$ rad.

(or 86 complete revolutions).

1.4. the angular velocity of the shaft at $t=0$ is

$$\omega(0) = 241 \text{ rad s}^{-1}.$$

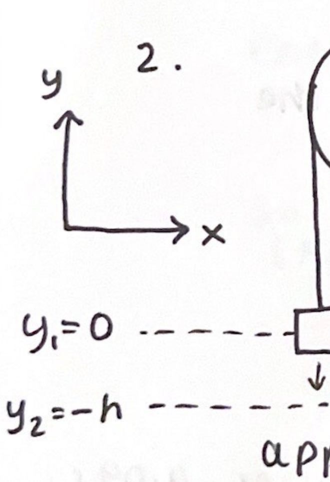
1.5. the average angular velocity is

$$\omega_{av.} = \frac{\theta(t_2) - \theta(t_1)}{t_2 - t_1}$$

$$= \frac{544.5 - 0}{4.09 - 0}$$

$$\Rightarrow \omega_{av.} = 133 \text{ rad s}^{-1}.$$

2.



apply conservation of energy. For the uniform solid disc, its kinetic energy under rotational motion is

$$T_p = T_{pulley} = \frac{1}{2} I_p \omega^2,$$

where $I_p = \frac{1}{2} m_p r_p^2$ is the moment of inertia and ω is the angular velocity of the pulley. the speed of the stone is related to the angular velocity of the pulley via

$$v = r_p \omega .$$

- the moment of inertia of the pulley is

$$I_p = \frac{1}{2} m_p r_p^2 = \frac{1}{2}(2.90)(0.100)^2$$

$$\Rightarrow I_p = 0.0145 \text{ kg m}^2$$

- the kinetic energy of the pulley is given as $T_p = 3.30$ J.

- the angular velocity of the pulley is

$$\omega = \sqrt{\frac{2T_p}{I_p}} = \sqrt{\frac{2(3.30)}{0.0145}} = 21.3 \text{ rad s}^{-1}.$$

- the stone thus has speed

$$V_s = r_p \omega = 0.100 \cdot 21.3 = 2.13 \text{ ms}^{-1}$$

So its kinetic energy is

$$T_{stone} = T_s = \frac{1}{2} m_s v_s^2 = \frac{1}{2}(1.60)(2.13)^2$$

$$\Rightarrow T_s = 3.63 \text{ J}.$$

- we have all our kinetic energies, so apply consv^n of energy:

$$\underbrace{T_i + U_i}_{\substack{=0 \text{ as released from rest } (T_i = 0) \\ \text{and at 0 height } (U_i = 0)}} = \underbrace{T_f + U_f}_{= T_p + T_s + m_s g(-h)} .$$

$\begin{cases} i = \text{initial, } t=0 \\ f = \text{final, at height } h \end{cases}$

hence

$$0 = T_p + T_s + mg(-h)$$

$$\Rightarrow h = \frac{T_p + T_s}{mg}$$

$$\Rightarrow h = \frac{3.30 + 3.63}{1.60(9.81)} = 0.442\ m$$

2.2. the pulley's kinetic energy T_p accounts for

$$\frac{T_p}{T_p + T_s} = \frac{3.30}{3.30 + 3.63} = 0.476$$

47.6 % of the total kinetic energy.

3.1 If we scale the length scales by f, the volume & mass are scaled by f^3.
the moment of inertia scales as

$$I \rightarrow \sum_i \left[m_i (f^3) \cdot r_i (f)^2\right] = \left(\sum_i m_i r_i\right) f^5 .$$

$\Rightarrow$ I scales as f^5.

3.2. the scaling factor is 48 as we are increasing the lengths.
Since the rotational kinetic energy is directly proportional to the moment of inertia, $T \propto I$, $T_{true} = f^5 T_{model}$

$$\Rightarrow T_{true} = 48^5 (2.50) = 6.4 \times 10^8\ J.$$